

Diffraction at small M^2/Q^2 in the QCD dipole picture

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Abstract. Using the QCD dipole picture of BFKL dynamics, the quasi-elastic component of the diffractive γ^* -dipole cross-section, dominating at small M^2/Q^2 , is calculated.

1 Introduction

Diffraction dissociation of virtual photons, a process on the border line between the perturbative and non-perturbative regions, represents a challenging problem for quantum chromodynamics [1]. It is therefore of no surprise that it became a subject of numerous studies by various methods [2]–[12].

The present investigation continues our effort [6]–[8] to reach a unified description of both total and diffractive cross-sections of virtual photons in the framework of the QCD dipole picture of high-energy interactions [9, 10]. More precisely, it is an attempt to obtain, within the approximations inherent to the QCD dipole model, an exact formula for the quasi-elastic component [6] of the (virtual) photon diffractive dissociation at arbitrary fixed momentum transfer. It is a generalization of a recent calculation [8] of the forward photon diffractive processes. The need for an *exact* formula is emphasized by (i) recent success of the approximate dipole model description [11, 12] of the HERA data [13] and (ii) improved accuracy of recent measurements [14] which provide the first results on momentum transfer dependence of diffractive structure functions.

We restrict ourselves to the so-called quasi-elastic component of the diffractive cross-section, dominating at small M^2/Q^2 , where M is the mass of the diffractive system produced and Q is the virtuality of the photon. The calculation of the high mass component at non-zero momentum transfer was presented in [7].

In the QCD dipole picture the cross-section for the quasi-elastic scattering of a virtual photon on a single dipole target can be written as [6]

$$\frac{d\sigma}{dM^2 d^2 p_T} = \frac{2N_c}{4\pi^2} \int d^2 k \left| \langle \vec{k}, M^2 | T_{p_T}^{\text{qel}} | Q \rangle \right|^2. \quad (1)$$

Here p_T is the transverse momentum after the collision. The transverse vector $\vec{k} = z\vec{k}_1 - (1-z)\vec{k}_2 = (|k|, \phi_k)$, is such that $\vec{k}^2 = M^2 z(1-z)$ in the impact parameter

approximation¹. By convention, the quark 3-momentum is $((1-z)E_\gamma, \vec{k}_1)$ and the antiquark (zE_γ, \vec{k}_2) . In (1), one has

$$\langle \vec{k}, M^2 | T_{p_T}^{\text{qel}} | Q \rangle = \int_0^1 \int d^2 r \langle \vec{k}, M^2 | \vec{r}, z \rangle \langle \vec{r}, z | T_{p_T} | \vec{r}, z \rangle \times \Psi(\vec{r}, z; Q). \quad (2)$$

Here $\vec{r} = (|r|, \phi_r)$ is the relative transverse distance in the $(q\bar{q})$ pair, z is the light-cone momentum fraction of one of the quarks and $\langle \vec{r}, z | T_{p_T} | \vec{r}, z \rangle$ is the elastic amplitude for scattering of the dipole of transverse size r on the target dipole of size r_0 . $\Psi(\vec{r}, z; Q)$ are the light-cone photon wave functions [15].

2 Calculations

Following the argument in [6] (c.f. also [8]), we can write (1) in the form

$$\frac{d\sigma}{dM^2 d^2 p_T} = \frac{2N_c}{4\pi^2} \int_0^1 dz z(1-z) \frac{1}{2} d\phi_k \left| G(\hat{M}, z, p_T; x_{\mathcal{P}}) \right|^2, \quad (3)$$

where

$$G(k, z, p_T; x_{\mathcal{P}}) = \frac{1}{2\pi} \int d^2 r e^{i\hat{M}r} \langle \vec{r}, z | T_{p_T} | \vec{r}, z \rangle \times \Psi(r, z; \hat{Q}) \quad (4)$$

¹ The formula (1) was derived in [6] in the impact parameter approximation. Consequently it is valid only in the limit $\frac{p_T}{E_\gamma} \equiv \frac{2m_p p_T}{Q^2} x \ll 1$, appropriate at small x . Indeed, in this limit, there is no difference between the initial and final transverse plane.

and \hat{M} is the vector of length

$$\hat{M} = M\sqrt{z(1-z)} \quad (5)$$

parallel to \vec{k} . Equation (4) is the starting point of our calculation.

The difficult part of the task is to perform the integration over d^2r in (4). To this end we need an adequate formula for the dipole–dipole amplitude $\langle \vec{r}, z | T_{p_T} | \vec{r}, z \rangle$. For $p_T = 0$ an explicit formula is available [9] and this allowed the calculation of [8]. For $p_T \neq 0$, however, the calculation is much more involved. In this paper we perform it using the methods developed recently in [16].

The starting point is the general expression [17]

$$\langle \vec{r}, z | T_{p_T} | \vec{r}, z \rangle = 4\pi\alpha^2 \sum_{n=-\infty}^{n=\infty} \int \frac{d\nu}{\pi} e^{\omega(n,\nu)Y} d_{n,\nu} \times |r| E_{p_T}^{n,\nu}(r) |r_0| \bar{E}_{p_T}^{n,\nu}(r_0), \quad (6)$$

where

$$d_{n,\nu} = \left\{ 16 \left[\nu^2 + \left(\frac{n-1}{2} \right)^2 \right] \left[\nu^2 + \left(\frac{n+1}{2} \right)^2 \right] \right\}^{-1}, \quad (7)$$

$$\omega(n,\nu) = \frac{2\alpha N_c}{\pi} \times \left\{ \psi(1) - \text{Re} \left[\psi \left(\frac{1}{2} (|n| + 1) + i\nu \right) \right] \right\} \quad (8)$$

and $E_{p_T}^{n,\nu}(r)$ are eigenfunctions of the conformal operator in the mixed representation defined in [17]. The explicit expression for $E_{p_T}^{n,\nu}(r)$ is fairly complicated [16] but, fortunately, we shall not need it here. We quote, for future reference, only the formula for $E_0^{n,\nu}(r)$:

$$E_0^{n,\nu}(r) = |r|^{-2i\nu} e^{in\phi_r}. \quad (9)$$

The sum over n in (6) was introduced for technical reasons. In the high-energy (large Y) limit, the term with $n = 0$ dominates.

To proceed, it is useful at this point to introduce the explicit formulae for $\Psi(\vec{r}, z; Q)$. We write them in the form

$$\Psi(\vec{r}, z; Q) = C \hat{Q} \Phi_{T,L}(z) \chi_{T,L}(r, \hat{Q}) \quad (10)$$

with $\hat{Q} = Q\sqrt{z(1-z)}$, $C = \sqrt{\alpha_{\text{em}}} e_{(f)}/2\pi$ ($e_{(f)}$ is the charge of a quark) and

$$\begin{aligned} \Phi_T &= z; & \Phi_L &= 2\sqrt{z(1-z)}; \\ \chi_T &= e^{i\phi_r} K_1(\hat{Q}r); & \chi_L &= K_0(\hat{Q}r). \end{aligned} \quad (11)$$

The subscripts T and L denote transverse (right-handed) and longitudinal polarizations of the incident photon (for left-handed photons one should replace z by $1-z$). Using (10), one obtains

$$\begin{aligned} G_{T,L}(k, z, p_T; x_{\mathcal{P}}) &= D |r_0| \hat{Q} \Phi_{T,L}(z) \\ &\times \sum_n \int \frac{d\nu}{\pi} d_{n,\nu} e^{\omega(n,\nu)Y} E_{p_T}^{n,\nu}(r_0) g_{T,L}^{n,\nu}, \end{aligned} \quad (12)$$

where $D \equiv 4\pi\alpha^2 C = 2\alpha^2 \sqrt{\alpha_{\text{em}}} e_{(f)}$,

$$g_{T,L}^{n,\nu}(\hat{Q}, \hat{M}) = \frac{1}{2\pi} \int d^2r \psi_{T,L}(r, \hat{Q}, \hat{M}) E_{p_T}^{n,\nu}(r) \quad (13)$$

and

$$\psi_{T,L}(r, \hat{Q}, \hat{M}) = e^{i\hat{M}r} \chi_{T,L}(r, \hat{Q}) |r|. \quad (14)$$

We can now expand $\psi_{T,L}(r, \hat{Q}, \hat{M})$ in terms of conformal eigenfunctions at $p_T = 0$:

$$\psi_{T,L}(r, \hat{Q}, \hat{M}) = \sum_n \int d\nu \psi_{T,L}^{n,\nu}(\hat{Q}, \hat{M}) \frac{E_0^{n,\nu}(r)}{|r|^2}. \quad (15)$$

The inverse transform is

$$\psi_{T,L}^{n,\nu}(\hat{Q}, \hat{M}) = \frac{1}{2\pi^2} \int d^2r \psi_{T,L}(r, \hat{Q}, \hat{M}) E_0^{n,\nu}(r) \quad (16)$$

and thus we obtain

$$g_{T,L}^{n,\nu}(\hat{Q}, \hat{M}) = \frac{1}{2\pi} \sum_{n'} \int d\nu' \psi_{T,L}^{n',\nu'}(\hat{Q}, \hat{M}) \mathcal{I}_{p_T}^{n',\nu,\nu'}. \quad (17)$$

We see that the integration over r is reduced to

$$\mathcal{I}_{p_T}^{n',\nu,\nu'} \equiv \int d^2r \frac{E_{p_T}^{n,\nu}(r) E_0^{n',\nu'}(r)}{|r|^2}, \quad (18)$$

which was explicitly calculated in [16] with the result

$$\begin{aligned} \mathcal{I}_{p_T}^{n',\nu,\nu'} &= \frac{\pi}{2} (-1)^{(n-n')/2} \left[\frac{p_T}{8} \right]^{\tilde{\mu}-\tilde{\mu}'} \left[\frac{\bar{p}_T}{8} \right]^{\mu-\mu'} \\ &\times \frac{\Gamma(1-\mu)}{\Gamma(\tilde{\mu})} \frac{\Gamma(\mu+\mu'/2) \Gamma(-\mu+\mu'/2)}{\Gamma(1-(\tilde{\mu}+\tilde{\mu}')/2) \Gamma(1-(\tilde{\mu}'-\tilde{\mu})/2)}, \end{aligned} \quad (19)$$

for $n-n'$ even and 0 for $n-n'$ odd. Here $\mu = i\nu - n/2$, $\tilde{\mu} = i\nu + n/2$.

We can now calculate $\psi^{n,\nu}(\hat{Q}, \hat{M})$. From (16) and (9) we have

$$\psi_{T,L}^{n,\nu}(\hat{Q}, \hat{M}) = \frac{1}{2\pi^2} \int d^2r \psi_{T,L}(r, \hat{Q}, \hat{M}) |r|^{-2i\nu} e^{in\phi_k}, \quad (20)$$

where ϕ_k is the r azimuthal angle with respect to p_T .

Using

$$\int d\Psi e^{i\hat{M}\rho \cos \Psi \pm im\Psi} \equiv 2\pi e^{im\pi/2} J_m(\hat{M}\rho), \quad (21)$$

we find for right-handed photons (for left-handed photons, $n+1$ should be replaced by $n-1$)

$$\begin{aligned} \psi_T^{n,\nu}(\hat{Q}, \hat{M}) &= \frac{1}{\pi} \int r^2 dr r^{-2i\nu} \exp \left(i(n+1) \frac{\pi}{2} - i\phi_k \right) \\ &\times J_{n+1}(\hat{M}r) K_1(\hat{Q}r). \end{aligned} \quad (22)$$

For the longitudinal photons we obtain

$$\psi_L^{n,\nu}(\hat{Q}, \hat{M}) = \frac{1}{\pi} \int r^2 dr r^{-2i\nu} e^{in\pi/2} J_n(\hat{M}r) K_0(\hat{Q}r) \quad (23)$$

Using well-known relations, we have

$$\begin{aligned} & \int_0^\infty d\rho \rho^{-\lambda} K_h(\hat{Q}\rho) J_l(\hat{M}\rho) \\ &= \frac{1}{4} \left(\frac{M}{Q}\right)^l \left(\frac{\hat{Q}}{2}\right)^{\lambda-1} \\ & \times \frac{\Gamma\left(\frac{1-\lambda+l+h}{2}\right) \Gamma\left(\frac{1-\lambda+l-h}{2}\right)}{\Gamma(l+1)} \\ & \times {}_2F_1\left(\frac{1-\lambda+l+h}{2}, \frac{1-\lambda+l-h}{2}, l+1; -\frac{M^2}{Q^2}\right) \end{aligned} \quad (24)$$

and

$$\begin{aligned} & {}_2F_1\left(\frac{1-\lambda+l+h}{2}, \frac{1-\lambda+l-h}{2}, l+1; -\frac{M^2}{Q^2}\right) \\ &= (\beta)^{\frac{1-\lambda+l-h}{2}} \\ & \times {}_2F_1\left(\frac{1-\lambda+l-h}{2}, \frac{1+\lambda+l-h}{2}, l+1; 1-\beta\right) \end{aligned} \quad (25)$$

where h and l are positive integers; thus we find

$$\begin{aligned} & \psi_{\text{T,right}}^{n,\nu}(\hat{Q}, \hat{M}) \\ &= \frac{1}{4\pi} \exp\left(-i\pi \frac{n+1}{2} - i\phi_k\right) \left(\frac{M}{Q}\right)^{|n+1|} \left(\frac{\hat{Q}}{2}\right)^{2i\nu-3} \\ & \times \frac{\Gamma(-i\nu+2+|n+1|/2) \Gamma(-i\nu+1+|n+1|/2)}{\Gamma(|n+1|+1)} \\ & \times (\beta)^{-i\nu+1+|n+1|/2} \\ & \times {}_2F_1\left(-i\nu+1+\frac{|n+1|}{2}, \frac{|n+1|}{2}, \right. \\ & \left. +i\nu-1, |n+1|+1; 1-\beta\right), \end{aligned} \quad (26)$$

$$\begin{aligned} & \psi_L^{n,\nu}(\hat{Q}, \hat{M}) \\ &= \frac{1}{4\pi} \exp\left(i\pi \frac{n}{2} - i\phi_k\right) \left(\frac{M}{Q}\right)^{|n|} \left(\frac{\hat{Q}}{2}\right)^{2i\nu-3} \\ & \times \frac{\Gamma^2(-i\nu+3/2+|n|/2)}{\Gamma(|n|+1)} (\beta)^{-i\nu+3/2+|n|/2} \\ & \times {}_2F_1\left(-i\nu+\frac{3}{2}+\frac{|n|}{2}, \frac{|n|}{2}, \right. \\ & \left. +i\nu-\frac{1}{2}, |n|+1; 1-\beta\right). \end{aligned} \quad (27)$$

Next we write

$$\begin{aligned} r_{\text{T}}(\nu_1, \nu_2) &= 2B(-i\nu_2 + i\nu_1 + 2, -i\nu_2 + i\nu_1), \\ r_{\text{L}}(\nu_1, \nu_2) &= 4B(-i\nu_2 + i\nu_1 + 1, -i\nu_2 + i\nu_1 + 1), \end{aligned} \quad (28)$$

which are factors coming from integrals over the variable z (the factor 2 in r_{T} comes from summing over photon

helicities). All in all, we obtain

$$\begin{aligned} & \frac{d\sigma}{dM^2 d^2 p_{\text{T}}} \\ &= \frac{N_c}{2\pi} \int_0^1 dz z(1-z) \left|G(\hat{M}, z; x_{\mathcal{P}})\right|^2 \\ &= \frac{N_c}{8\pi} Q^2 D^2 \\ & \times \sum_{n_1} \int \frac{d\nu_1}{\pi} e^{\omega(n_1, \nu_1)} Y_{d_{n_1, \nu_1}} |r_0| \bar{E}_{p_{\text{T}}}^{n_1, \nu_1}(r_0) \\ & \times \sum_{n_2} \int \frac{d\nu_2}{\pi} e^{\omega(n_2, \nu_2)} Y_{\bar{d}_{n_2, \nu_2}} |r_0| E_{p_{\text{T}}}^{n_2, \nu_2}(r_0) \\ & \times \sum_{n'_1} \int \frac{d\nu'_1}{\pi} \psi_{\text{T,L}}^{n'_1, \nu'_1}(Q, M) \mathcal{I}_{P_{\text{T}}}^{n_1, n'_1, \nu_1, \nu'_1} \\ & \times \sum_{n'_2} \int \frac{d\nu'_2}{\pi} \bar{\psi}_{\text{T,L}}^{n'_2, \nu'_2}(Q, M) \bar{\mathcal{I}}_{P_{\text{T}}}^{n_2, n'_2, \nu_2, \nu'_2} r_{\text{T,L}}(\nu'_1, \nu'_2). \end{aligned} \quad (29)$$

This completes the calculation.

Note that at $p_{\text{T}} = 0$, we have

$$I_0^{nn', \nu\nu'} = 2\pi^2 \delta_{nn'} \delta(\nu - \nu') \quad (30)$$

and thus, using (9), we obtain

$$\begin{aligned} & \frac{d\sigma}{dM^2 d^2 p_{\text{T}}} \Big|_{p_{\text{T}}=0} = 8\pi \alpha_{\text{em}} \alpha^4 N_c e_f^2 \left(\frac{Qr_0}{2}\right)^2 \\ & \times \sum_{n_1} \left(\int \frac{d\nu_1}{\pi} e^{\omega(n_1, \nu_1)} Y_{d_{n_1, \nu_1}}(r_0)^{2i\nu_1} \psi_{\text{T,L}}^{n_1, \nu_1}(Q, M)\right) \\ & \times \sum_{n_2} \left(\int \frac{d\nu_2}{\pi} e^{\omega(n_2, \nu_2)} Y_{\bar{d}_{n_2, \nu_2}}(r_0)^{-2i\nu_2} \bar{\psi}_{\text{T,L}}^{n_2, \nu_2}(Q, M) r_{\text{T,L}}(\nu_1, \nu_2)\right), \end{aligned} \quad (31)$$

which, for $n_1 = n_2 = 0$, is identical to the result obtained in [8].

3 Results

To obtain more insight into the P_{T} dependence of the cross-section given by (29), it is useful to evaluate the integrals over ν'_1 and ν'_2 in terms of residues of the relevant poles of the integrands. To this end we observe that the convergence properties of these integrals are determined by the factor

$$\left(\frac{1}{\hat{p}_{\text{T}}}\right)^{2i\nu'} \equiv \left(\frac{4Q}{P_{\text{T}}\sqrt{\beta}}\right)^{2i\nu'}. \quad (32)$$

This factor is easily identified when the explicit expressions (19), (22) and (23) are introduced into the product $\Psi_{\text{T,L}}^{n', \nu'}(Q, M) \mathcal{I}_{P_{\text{T}}}^{n, n', \nu, \nu'}$. Thus for

$$\hat{p}_{\text{T}} \equiv \frac{P_{\text{T}}\sqrt{\beta}}{4Q} < 1, \quad (33)$$

the contour integrals in ν'_1 and ν'_2 must be chosen in the lower half of the complex plane, so only the poles at $\Re(i\nu') \leq 0$ contribute. The integrals are given by the residues of the *moving* poles $i\nu' = \pm i\nu - 2p$ leading to an expansion in terms of powers $(\hat{p}_T)^{4p}$ of the “reduced” transverse momentum (33). Thus one obtains

$$\begin{aligned} \frac{d\sigma}{dM^2 d^2 P_T} &= 8\pi\alpha_{em}\alpha^4 N_c e_f^2 \left(\frac{Qr_0}{2}\right)^2 \\ &\times \sum_{n_1} \left(\int \frac{d\nu_1}{\pi} e^{\omega(n_1, \nu_1) Y} d_{n_1, \nu_1} \right) \\ &\times \bar{E}_{P_T}^{n_1, \nu_1}(r_0) \psi_{T,L}^{n_1, \nu_1}(Q, M) \\ &\times \sum_{n_2} \left(\int \frac{d\nu_2}{\pi} e^{\omega(n_2, \nu_2) Y} \bar{d}_{n_2, \nu_2} \right) \\ &\times E_{P_T}^{n_2, \nu_2}(r_0) \bar{\psi}_{T,L}^{n_2, \nu_2}(Q, M) r_{T,L}(\nu_1, \nu_2) \\ &\times \left[1 + \mathcal{O}(\hat{p}_T)^4 \right]. \end{aligned} \quad (34)$$

On the other hand, for larger P_T ,²

$$\hat{p}_T \equiv \frac{P_T \sqrt{\beta}}{4Q} > 1, \quad (35)$$

the contour integrals must be closed on the upper half of the complex plane. Consequently, only poles of the Ψ functions at $\Re(i\nu') > 0$ contribute. The integrals are thus given by *fixed* poles at $i\nu' = 3/2 + 2p$ and $i\nu' = 5/2 + 2p$ leading to an expansion independent of ν , resulting in a series expansion in powers of $1/\hat{p}_T$. The two regimes are thus governed by different types of singularities, as was already noticed about vertices of BFKL pomerons [16,18].

4 Concluding remarks

(a) The presented calculation extends the results in [8] to non-vanishing momentum transfer and thus, together with [7], where the large mass component was calculated, it completes the derivation of the hard diffractive cross-section in the QCD dipole picture.

(b) The p_T dependence in (29) and (34) may be modified by the proton form factor and possibly other non-perturbative effects. As the same effects would operate also for the large mass component in [7], one may hope that the relative weight of the two components is reasonably well described by our result. A comparison with data should therefore be a significant test of the dipole approach. It would be particularly interesting to compare this new result with the satisfactory description of data obtained in an approximate version of the dipole model [12].

(c) Our formulae (29) and (34) sum over all conformal spin values n allowed by selection rules of the BFKL vertices [16]. At high energies the term with $n = 0$, corresponding to the so-called hard pomeron [1], dominates.

Consequently the phenomenological discussion is usually restricted to $n = 0$. As argued recently [19], however, the analysis of total cross-section data [20] allows the interpretation of the next term in the expansion, namely $n = 2$, as being at the origin of the so-called soft pomeron [21]. Our formula can thus provide an independent test of this hypothesis in hard diffraction dissociation.

(d) It would be interesting to verify if the results can be directly derived from Feynman diagrams, as is the case for the high mass diffraction processes [2].

Using the QCD dipole picture of BFKL dynamics we have calculated the quasi-elastic component of the diffractive γ^* -dipole cross-section, dominating at small M^2/Q^2 . This work, together with [7], completes the calculation of hard diffraction in the framework of the QCD dipole model and thus allows an extension of the previous phenomenological analyses based on approximate formulae [11,12]. This should in turn allow perturbative and non-perturbative QCD contributions to the diffraction on the proton target to be separated. Finally, let us note that the formulae obtained in [7] and here can easily be generalized to $\gamma^* - \gamma^*$ interactions and thus provide a basis for a phenomenological analysis of future collider data.

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² But not too large, see footnote 1.

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